Benefits of Permutation-Equivariance in Auction Mechanisms

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Motivation

Optimal auction mechanisms are assumed to be **permutation-equivariant** if the valuation distribution is invariant when permutating bidders and items.

Permutation over bidders and items

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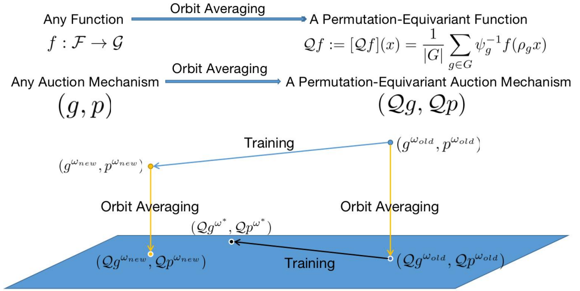
Permutation over allocation rule and payment rule

Problem

- What are the benefits to incorporate permutation-equivariance into auction mechanisms?
- How can permutation-equivariance help learn the (approximate) optimal auction mechanisms?

Studying Method

Orbit averaging can project any function into the equivariant function space, which enables us to extract the permutation-equivariant part of an auction mechanism and compare them.



Theoretical Benefits

In the setting of additive valuation and symmetric valuation,

- Permutation-equivariance **decreases the sum of all bidders' expected ex-post regrets** while maintaining the auctioneer's expected revenue, i.e.

$$\mathbb{E}_{(v,x,y)}\left[\sum_{i=1}^{n}[\mathcal{Q}p]_i(v,x,y)\right] = \mathbb{E}_{(v,x,y)}\left[\sum_{i=1}^{n}p_i(v,x,y)\right]$$

$$\mathbb{E}_{(v,x,y)}\left[\sum_{i=1}^{n}[\mathcal{Q}reg]_i(v,x,y)\right] \le \mathbb{E}_{(v,x,y)}\left[\sum_{i=1}^{n}reg_i(v,x,y)\right]$$

- Permutation-equivariance **improves the generalizability** and then **decreases the required sample complexity** for the desired generalization, proven by decreasing covering numbers.

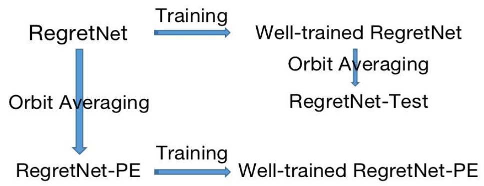
$$\mathcal{N}_{\infty,1}(\mathcal{QP}, r) \le \mathcal{N}_{\infty,1}(\mathcal{P}, r) \quad \text{and} \quad \mathcal{N}_{\infty,1}(\mathcal{QU}, r) \le \mathcal{N}_{\infty,1}(\mathcal{U}, r)$$

Implications

- Permutation-equivariance can help approach theoretical optimal **dominant strategy incentive compatible** (DSIC) condition.
- Permutation-equivariance promises **a larger expected revenue** when the sum of all bidder's expected ex-post regrets is fixed.
- An extra positive term existing in the expected ex-post regret penalizes the "non-permutation-equivariance" and affects the learning performance.
- Orbit averaging can serve as a plug-and-play method to improve any non-permutation-equivariant auction mechanism.
- Orbit averaging can be used to design new permutation-equivariant architectures with proven better generalizability.

Experiments

We design permutation-equivariant versions of RegretNet (RegretNet-PE and RegretNet-test) by projecting the RegretNet to the permutation-equivariant mechanism space in the training stage and test stage, respectively.



Experimental Results

We leverage the learned auction mechanism's expected **revenue**, **ex-post regret**, and the **generalization error** to evaluate the performance of the auction mechanism.

One-item:

Method	2 × 1 Uniform			3 × 1 Uniform			5 × 1 Uniform		
	Revenue	Regret	GE	Revenue	Regret	GE	Revenue	Regret	GE
Optimal	0.417	0	-	0.531	0	-	0.672	0	-
RegretNet	0.415	0.00017	0.00006	0.535	0.00018	0.00011	0.658	0.00016	0.00006
RegretNet-Test	0.415	0.00016	-	0.535	0.00013	-	0.658	0.00006	-
RegretNet-PE	0.420	0.00014	0.00004	0.541	0.00016	0.00010	0.677	0.00013	0.00005

Two-item:

Method	1 × 2 Uniform		2 × 2 Uniform	
	Revenue	Regret	Revenue	Regret
RegretNet	0.562	0.00061	0.870	0.00070
EquivariantNet	0.551	0.00013	0.873	0.00100
RegretNet-Test	0.562	0.00052	0.870	0.00054
RegretNet-PE	0.563	0.00037	0.913	0.00067